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Modeling heavy ion collisions in AdS/CFT

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Abstract: We construct a model of high energy heavy ion collisions as two ultrarelativistic shock waves colliding in AdS_5. We point out that shock waves corresponding to physical energy-momentum tensors of the nuclei completely stop almost immediately after the collision in AdS_5, which, on the field theory side, corresponds to complete nuclear stopping due to strong coupling effects, likely leading to Landau hydrodynamics. Since in real-life heavy ion collisions the large Bjorken x part of nuclear wave functions continues to move along the light cone trajectories of the incoming nuclei leaving the small-x partons behind, we conclude that a pure large coupling approach is not likely to adequately model nuclear collisions. We show that to account for small-coupling effects one can model the colliding nuclei by two (unphysical) ultrarelativistic shock waves with zero net energy each (but with non-zero energy density). We use this model to study the energy density of the strongly-coupled matter created immediately after the collision. We argue that expansion of the energy density ϵ in the powers of proper time τ squared corresponds on the gravity side to a perturbative expansion of the metric in graviton exchanges. Using such expansion we reproduce our earlier result [1] that the energy density of produced matter at mid-rapidity starts out as a constant (of time) in heavy ion collisions at large coupling.

Keywords: AdS-CFT Correspondence, Phenomenological Models.

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Contents

1. Introduction

There is a mounting phenomenological evidence coming from RHIC data for a strongly coupled medium created in heavy ion collisions [2 − 19]. Ideal hydrodynamics simulations have been extremely successful in describing data generated in heavy ion collisions at RHIC [2 − 9]. These analyses require very small shear viscosity [10], indicating that the medium is strongly coupled [11 − 15], and a very short thermalization time of the initially produced system, of the order of $0.3 \div 0.6$ fm/c [2 − 9]. At the same time, many bulk features of heavy ion collisions at RHIC which are sensitive to the initial-time dynamics, such as the energy, rapidity and centrality dependence of particle production are very well-described in the weakly-coupled framework of the Color Glass Condensate (CGC) [20 − 38] (for a review of CGC see [39 − 41]). In the CGC approach a heavy ion collision releases the small Bjorken-x partons in the nuclear wave functions, which quickly go on mass shell and become real: the resulting particle distribution in highly anisotropic in momentum space [42, 43]. A question then arises about how this initially weakly-coupled highly anisotropic system becomes isotropic and equilibrates very quickly, becoming a strongly-coupled, possibly thermal, medium.

So far, conventional perturbative descriptions of thermalization of the produced medium [44 − 47] have not been able to account for the short thermalization time of the order of a fraction of a fermi required by hydrodynamic simulations to describe the data. Perturbative thermalization in heavy ion collisions was also studied in [43, 48, 49], where it was concluded that perturbative thermalization scenarios are not likely to be compatible with RHIC data. This leads one to conclude that it is highly likely that non-perturbative large-coupling QCD effects are responsible for the apparent thermalization observed in RHIC data. While the research on perturbative thermalization scenarios along the lines outlined in [44 – 47] is vigorously pursued in the community, we believe it probable that the dynamics of the medium produced in a heavy ion collision at RHIC proceeds as follows. The medium starts out being weakly-coupled, being well-described by CGC. After a very short proper time, of the order of $\tau \gtrsim 1/Q_s \approx 0.2 \div 0.3\,\mathrm{fm/c}$ (and possibly at $\tau \approx 1/\Lambda_{\mathrm{QCD}} \approx 1\,\mathrm{fm/c}$) the coupling becomes strong. Strong coupling effects are likely to quickly thermalize the medium, allowing for its hydrodynamic description.

Unfortunately to date there is no consistent way to describe both the weakly-coupled and the strongly coupled dynamics of QCD medium in a unified framework. However, as we argued above, to understand the general physical nature of thermalization (and, more importantly, isotropization) of the medium, and to see whether isotropization and thermalization take place at all, a purely strong coupling approach seems, a priori, appropriate. Indeed at strong coupling QCD becomes non-perturbative and no controllable dynamical calculation appears to be possible. Instead one could use the Anti-de Sitter space/conformal field theory (AdS/CFT) correspondence [50 – 53] to understand the same process for $\mathcal{N} = 4$ super-Yang-Mills theory. This correspondence, and in particular the gauge-gravity duality which follows from it, allows one to understand strong coupling effects in such QCD-like theories as $\mathcal{N} = 4$ super-Yang-Mills theory using super-gravity in 5 dimensions. AdS/CFT correspondence has been useful in providing insight in the behavior of the shear viscosity in strongly-coupled gauge theories [11 – 15] along with providing other interesting results on the evolution of the medium created in heavy ion collisions [54 – 67, 1].

The goal of this paper is to make progress in constructing a dual geometry in AdS_5 space for heavy ion collisions with the goal of understanding the onset of isotropization and thermalization of the produced medium. The previous paper on the subject by two of the authors [1] studied the very early time dynamics of the medium produced in the collisions. It was shown that, assuming rapidity-independence of the produced medium and assuming non-negativity of its energy density, one would obtain that the energy density of the produced medium should start out as a constant of time at very early times immediately after the collisions. This implied that the longitudinal pressure of this early-time medium is negative and the medium is thus highly anisotropic. This behavior is similar to that of the weakly-coupled CGC medium at early times [32, 68, 69]. The problem of isotropization and the onset of Bjorken hydrodynamics [70] in this framework can be formulated as the question about understanding the transition from the negative longitudinal pressure of the medium at early times to the positive longitudinal pressure (comparable to the transverse pressure) at late times [43]. In [54, 56, 71] it was shown that the dynamics of a strongly-coupled rapidity-independent medium leads to Bjorken hydrodynamics behavior at late proper times: it is therefore likely that isotropization transition takes place at some intermediate time between the early-time dynamics of [1] and the late time dynamics

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of [54, 56, 71].

To better understand this transition one needs to find the energy-momentum tensor of the medium at a later times than considered in [1]. Unfortunately, a consistent expansion of the energy-momentum tensor in the powers of proper time τ requires some knowledge of the geometry dual to the colliding nuclei. (In [1] nothing was assumed about the colliding nuclei, except that they lead to rapidity-independent distribution of matter.) Thus in this paper we try to construct a geometry dual to the collision of the two nuclei. We model two nuclei as shock waves in AdS_5. Modeling nuclei with shock waves has previously been considered in [66] for AdS_3 corresponding to gauge theory in two space-time dimensions and in [61, 67] for AdS_5.

The paper is organized as follows. We begin by spelling out some general formulas used in the paper in section 2. We proceed by setting up the problem in section 3. We start with two colliding shock waves, each given by the metric like that shown in eq. (3.1) (see [54]). We argue that this metric corresponds to a single graviton field, as shown in figure 2. We then argue that a consistent expansion of the metric at the time after the collision can be constructed by considering higher order graviton exchanges between the boundary and the bulk, as shown in figure 3. We construct a general next-to-leading order perturbative contribution to the metric in graviton exchanges in section 4. The solution is given by eqs. (4.8), (4.16), (4.17), (4.18), (4.19), (4.20), (4.21), (4.22), (4.23).

Using the obtained solution we study the collision of two physical shock waves in section 5. We conclude that the shock waves, and the nuclei in the boundary theory, completely stop very shortly after the collision, after a time of the order of inverse typical transverse momentum scale in the problem, as given in eq. (5.19). We interpret this result as creation of a black hole, similar to what is suggested for collisions of particles at transplanckian energies [72 – 74], though our black hole would be created in the bulk. On the gauge theory side this implies that strong coupling effects would completely stop the nuclei shortly after the collision, on the time scale less than or equal to 1 fm/c. Such strong coupling effects are likely to thermalize the system soon after the stopping, leading to Landau hydrodynamic description of the system [75]. This thermalization scenario is very different from the onset of Bjorken hydrodynamics outlined above. It is possible in principle and has been advocated in the literature [76].

However, we believe that the Landau hydrodynamic scenario is not likely to be relevant for heavy ion collisions. This claim is supported by the following observations. On the one hand, the ideal Bjorken hydrodynamics has been extremely successful in describing the particle spectra and elliptic flow [2 – 9]. On the other hand, the agreement with experimental data of the predictions based on the weakly-coupled CGC approaches to description of particle multiplicities in nuclear collisions [33 – 35] indicates that the complete nuclear stopping does not happen in the actual collisions. Indeed, in high-energy scattering at small coupling the hard (large Bjorken-x) parts of the nuclear wave functions simply go through each other without recoil. In the quasi-classical CGC limit this leads to rapidity-independent, Bjorken-like picture of particle production in heavy ion collisions [26, 77, 27, 78, 79]. Thus it appears that weak coupling effects are a *key* ingredient for a proper description of the space-time structure of heavy ion collisions, even in the medium becomes strongly coupled shortly after the collision.

As we do not know how to model weak coupling effects in AdS/CFT, we try in section 6 to mimic them by introducing unphysical shock waves with non-positive-definite energy density. Such shock waves are indeed unphysical and can not follow from the underlying string theory. Even for the gravity in the bulk these shock waves can only serve as sources external to the theory. An example of the energy-momentum tensor of such shock wave is given in eq. (6.3). Using the general solution of section 4, we construct the early time energy density and pressures of the medium produced in the "collision" of two of such shock waves, which are shown in eq. (6.7). As can be seen from eq. (6.7), the energy density starts out as a constant in time. We have thus reproduced the results of [1], but now in a more dynamical setting. The problem of thermalization formulated with the help of these unphysical shock waves is probably that of isotropization of the produced medium to achieve Bjorken hydrodynamics [70], as described above and considered in [43, 1].

We discuss possible higher order corrections to the result of eq. (6.7) in section 7 and observe that a dilaton field may need to be introduced at higher orders to account for initial non-equilibrium between chromo-electric and chromo-magnetic modes in the medium (see eq. (7.6)). We conclude in section 8 by restating our main results.

Our solution found in section 4 is general and is valid for any shock wave profile, unlike the solution found in [67] which works only for delta-function shock waves. In the particular case of the delta-function shock waves, our solution reduces to that found in [67]. The general nature of our solution allowed us in section 5 to reach a more general physical conclusion that the collision of any two physical shock waves (with positive definite energy density) leads to complete stopping of the shock waves after the collision, probably leading to the formation of a black hole. In particular this conclusion applies to the delta-function shock waves used in [67]. Our solution also allows us to tackle unphysical shock waves in section 6, which are impossible to handle in the formalism of [67]. We should also note that the stopping of physical shock waves found here does not happen in AdS_3 (see [66]), since in 1+1 dimensional gauge theory the nuclei are point particles and stopping for them is impossible, as there is no transverse directions in which the momentum could be channeled.

2. Some generalities

Throughout this paper we will work with the metric of AdS_5 written in terms of Fefferman-Graham coordinates [80]

$$ds^2 \;=\; \frac{L^2}{z^2}\,\left\{\tilde{g}_{\mu\nu}(x,z)\,dx^\mu\,dx^\nu + dz^2\right\} \tag{2.1}$$

where μ, ν run from 0 to 3 and z is the coordinate describing the 5th dimension. The boundary of the AdS space is at $z = 0$ and L is the curvature radius of the AdS space.

According to holographic renormalization [81], if one expands the 4-dimensional metric $\tilde{g}_{\mu\nu}(x,z)$ near the boundary of the AdS space

$$\tilde{g}_{\mu\nu}(x,z) \;=\; \tilde{g}^{(0)}_{\mu\nu}(x) + z^2\,\tilde{g}^{(2)}_{\mu\nu}(x) + z^4\,\tilde{g}^{(4)}_{\mu\nu}(x) + \ldots, \tag{2.2}$$

then, for Minkowski metric $\tilde{g}^{(0)}_{\mu\nu}(x) = \eta_{\mu\nu}$, one gets $\tilde{g}^{(2)}_{\mu\nu}(x) = 0$ and the expectation value of the energy-momentum tensor of the gauge theory is

$$\langle T_{\mu\nu} \rangle \; = \; \frac{N_c^2}{2\,\pi^2}\, \tilde{g}^{(4)}_{\mu\nu}(x). \tag{2.3}$$

Below we will use the light cone coordinates

$$x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}} \tag{2.4}$$

where x^3 is the collision axis of the two colliding nuclei. In these coordinates the empty AdS$_5$ metric is

$$ds^2 \; = \; \frac{L^2}{z^2} \left\{ -2\, dx^+ \, dx^- + dx_\perp^2 + dz^2 \right\}, \tag{2.5}$$

where $dx_\perp^2 = (dx^1)^2 + (dx^2)^2$ with x^1 and x^2 the transverse dimensions which we will denote using Latin indices, e.g. x^i. To describe nuclear collisions we will also use the proper time

$$\tau \; = \; \sqrt{2\, x^+ \, x^-} \tag{2.6}$$

and space-time rapidity

$$\eta \; = \; \frac{1}{2} \, \ln \frac{x^+}{x^-}. \tag{2.7}$$

Einstein equations in AdS$_5$ are

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + \Lambda_c \, g_{\mu\nu} = 0 \tag{2.8}$$

where $g_{\mu\nu}$ is the full 5-dimensional metric of the AdS$_5$ space, R is the scalar curvature and Λ_c is the cosmological constant. For AdS$_5$ we have

$$\Lambda_c \; = \; -\frac{6}{L^2}. \tag{2.9}$$

and eq. (2.8) gives

$$R \; = \; -\frac{20}{L^2}. \tag{2.10}$$

Eqs. (2.9), (2.10) yield

$$R_{\mu\nu} + \frac{4}{L^2}\, g_{\mu\nu} = 0. \tag{2.11}$$

Later in the paper we will also discuss the dynamics of a dilaton field φ coupled to gravity in AdS$_5$. In the presence of dilaton eq. (2.8) is modified to (see e.g. [82, 83, 71])

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + \Lambda_c \, g_{\mu\nu} \; = \; \frac{1}{2}\, \partial_\mu \varphi \, \partial_\nu \varphi - \frac{1}{4}\, g_{\mu\nu}\, \partial_\rho \varphi \, \partial^\rho \varphi. \tag{2.12}$$

eq. (2.12) can be simplified to give

$$R_{\mu\nu} + \frac{4}{L^2}\, g_{\mu\nu} \; = \; \frac{1}{2}\, \partial_\mu \varphi \, \partial_\nu \varphi. \tag{2.13}$$

The dilaton itself obeys the Klein-Gordon equation in curved space-time

$$\partial_\mu \left[\sqrt{-g}\, g^{\mu\nu}\, \partial_\nu \, \varphi \right] \; = \; 0. \tag{2.14}$$

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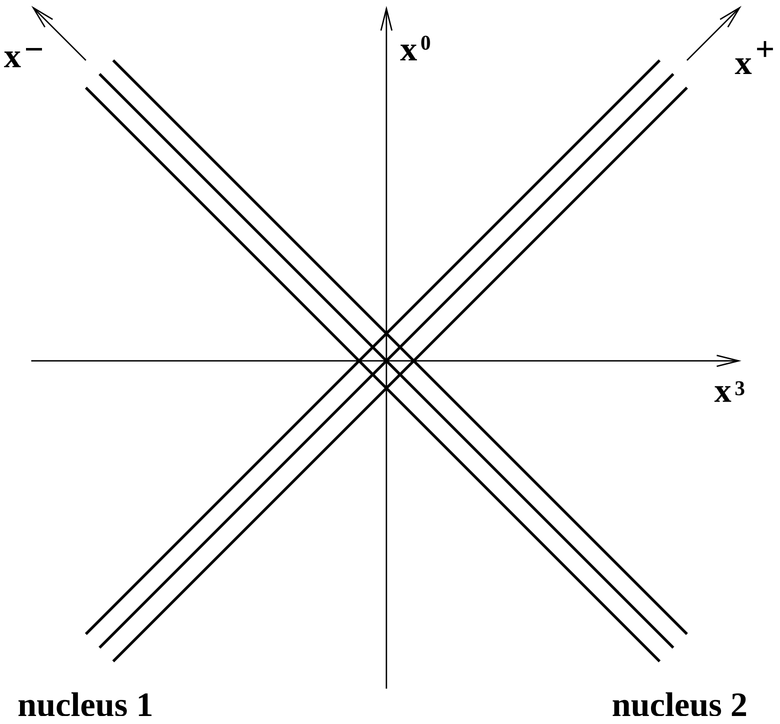

Figure 1: The space-time picture of the ultrarelativistic heavy ion collision in the center-of-mass frame. The collision axis is labeled x^3, the time is x^0.

3. Setting up the problem

Out goal is to construct a metric in AdS_5 which is dual to an ultrarelativistic heavy ion collision as pictured in figure 1. Throughout the discussion we will use Bjorken approximation of the nuclei having an infinite transverse extent [70] and being homogeneous (on the average) in the transverse direction, such that nothing in our problem would depend on the transverse coordinates x^1, x^2.

We start with a metric for a single ultrarelativistic nucleus moving along a light cone. As was noted by Janik and Peschanski [54] the following metric gives a solution of Einstein equations in AdS_5 in Fefferman-Graham coordinates [80]

$$ds^2 = \frac{L^2}{z^2} \left\{ -2\, dx^+ \, dx^- + \frac{2\,\pi^2}{N_c^2} \langle T_{--}(x^-) \rangle \, z^4 \, dx^{-\,2} + dx_\perp^2 + dz^2 \right\}. \tag{3.1}$$

eq. (3.1) is a solution of Einstein equations (2.11) for any expectation value of the energy-momentum tensor of the nucleus in four dimensions $\langle T_{--}(x^-) \rangle$, as long as it is a function of x^- only [54]. The factor of $2\,\pi^2/N_c^2$ is due to Newton's constant [81]. For an ultrarelativistic nucleus with infinite transverse extent moving along the x^+ axis (see figure 1) the leading components of the energy momentum tensor depend only on x^-. Hence the metric in eq. (3.1) adequately describes such a nucleus, though does not restrict the dependence of $\langle T_{--}(x^-) \rangle$ on x^-.

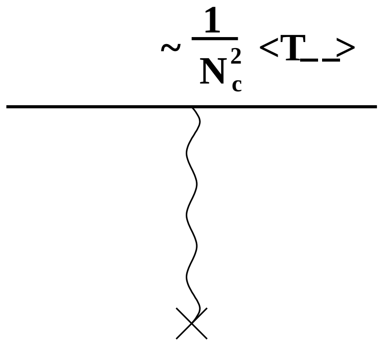

Figure 2: A representation of the metric (3.1) as a graviton (wavy line) exchange between the nucleus at the boundary of AdS space (the solid line) and the point in the bulk where the metric is measured (denoted by a cross).

While the metric (3.1) is an exact solution of the non-linear Einstein equations (2.11), it can also be represented perturbatively as a single graviton exchange between the source nucleus at the AdS boundary and the location in the bulk where we measure the metric/graviton field. This is shown in figure 2, where the solid line represents the nucleus and the wavy line is the graviton propagator. Incidentally a single graviton exchange, while being a first-order perturbation of the empty AdS space, is also an exact solution of Einstein equations. This means higher order tree-level graviton diagrams are zero. It is interesting to note that a similar property has been observed for gauge theories in covariant gauge [24, 84], where the exact solution of Yang-Mills equations with a single ultrarelativistic nucleus as a source is given by a single gluon exchange.

As one can see comparing the metric (3.1) with the diagram in figure 2, each graviton-nucleus vertex gives a factor

$$\sim \frac{1}{N_c^2} \langle T_{--}(x^-) \rangle. \tag{3.2}$$

If the nuclear energy-momentum tensor is N_c-independent, then in the large-N_c limit the factor in eq. (3.2) would be small and one could envision perturbative expansion in this parameter for the problem of collision of two nuclei. On the other hand, gauge-gravity duality is valid only in the large-N_c limit: hence we need $\langle T_{--}(x^-) \rangle \sim N_c^2$ to avoid having N_c-suppression for higher-order graviton exchanges. This could be achieved by imagining a nucleus with nucleons made out of $N_c^2 - 1$ "valence" gluons each. Then $\langle T_{--}(x^-) \rangle \sim N_c^2$ and multiple graviton exchanges will not be N_c-suppressed.

However, one may then worry that the expansion parameter also ceases to be small. Nevertheless it makes sense to expand in powers of $\langle T_{--}(x^-) \rangle$, as usually $\langle T_{--}(x^-) \rangle$ contains some momentum scale characterizing the nucleus such that one can keep track of the powers of this scale. Hence even if the expansion parameter is not small, the expansion is still well-defined and can be kept track of.

For instance, in the original proposal of Janik and Peschanski [54], the energy-momentum tensor due to valence quarks in the ultrarelativistic nucleus was taken to be

$$\langle T_{--}(x^-)\rangle \;=\; \mu\, N_c^2\, \delta(x^-) \tag{3.3}$$

with μ a scale having dimensions of mass cubed. Starting with the energy-momentum tensor of a single ultrarelativistic particle and performing the averaging along the lines shown in appendix A gives

$$\mu \propto p^+\, \Lambda^2\, A^{1/3} \tag{3.4}$$

where p^+ is the large light-cone momentum of the nucleons in the nucleus, A is the atomic number and Λ is some transverse momentum scale. In this case the expansion in powers of $\langle T_{--}(x^-)\rangle$ translates into the expansion in the powers of μ, which can be systematically resummed (see e.g. [67]).

Alternatively one could argue that at strong coupling the energy-momentum tensor is dominated not by valence quarks, but by the strong gluon fields of the nucleus. One can argue, based on conformal invariance, that the coordinate dependence of the energy-momentum tensor of such a strong gluon field in $\mathcal{N}=4$ SYM theory is the same as that of weakly coupled electromagnetic fields [85 – 87]. Performing a classical electrodynamics calculation of the energy-momentum tensor of a point charge, averaging over all transverse coordinates and summing over all nucleons yields (see appendix A for details)

$$\langle T_{--}(x^-)\rangle \;=\; \sqrt{\lambda}\,\Lambda^2\, N_c^2\, A^{1/3}\, \delta(x^{-\,2}) \tag{3.5}$$

with Λ some transverse momentum scale. At weak coupling in classical electrodynamics $\langle T_{--}(x^-)\rangle \sim g^2 \sim \lambda$ with λ the 't Hooft coupling

$$\lambda \;=\; g^2\, N_c. \tag{3.6}$$

Guided by the calculation of the heavy quark-antiquark potential at strong coupling in [88] we have replaced the coupling constant λ by $\sqrt{\lambda}$ in eq. (3.5). However, the exact power of λ in eq. (3.5) does not alter the subsequent discussion.

In case of the energy-momentum tensor in eq. (3.5) one can construct an expansion in the powers of Λ^2, which is again well-defined.

Using the perturbative expansion in the powers of the energy-momentum tensor, one can construct the metric dual to a heavy ion collision. At the lowest non-trivial order we begin by writing the metric as

$$\begin{aligned} ds^2 = \frac{L^2}{z^2}\Bigg\{ &-2\,dx^+\,dx^- + dx_\perp^2 + dz^2 + \frac{2\,\pi^2}{N_c^2}\,\langle T_{1\,--}(x^-)\rangle\, z^4\, dx^{-\,2} \\ &+ \frac{2\,\pi^2}{N_c^2}\,\langle T_{2\,++}(x^+)\rangle\, z^4\, dx^{+\,2} + \text{higher order graviton exchanges}\Bigg\} \end{aligned} \tag{3.7}$$

where $\langle T_{1\,--}(x^-)\rangle$ and $\langle T_{2\,++}(x^+)\rangle$ are the energy-momentum tensors of the colliding nuclei 1 and 2 as shown in figure 1. The metric in eq. (3.7) is that of two colliding shock waves in AdS_5. Higher order graviton exchanges will modify the shock waves after the collision and

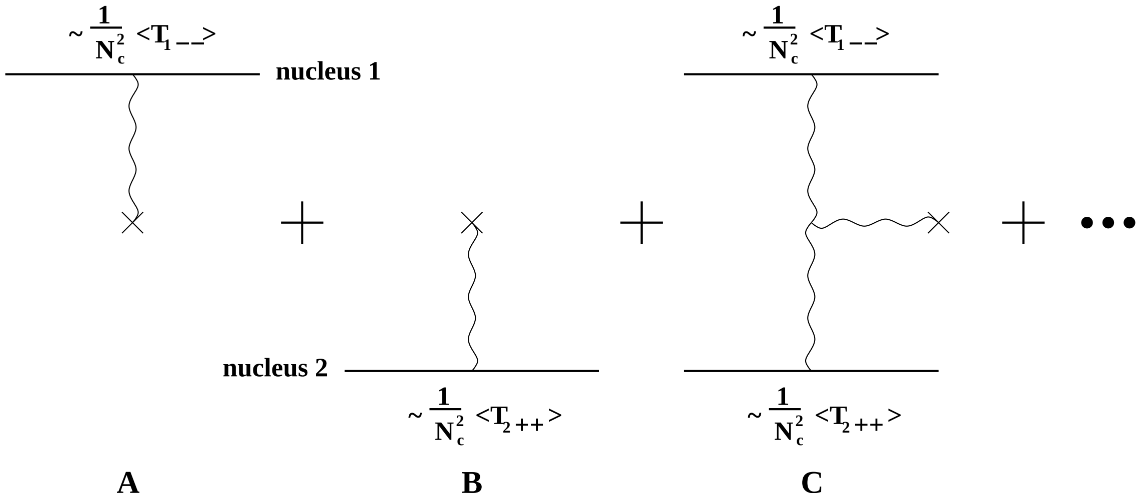

Figure 3: Diagrammatic representation of the metric in eq. (3.7). Wavy lines are graviton propagators between the boundary of the AdS space and the bulk. Graphs A and B correspond to the metrics of the first and the second nucleus correspondingly. Diagram C is an example of the higher order graviton exchange corrections. We calculate the contribution of this diagram below in section 4.

generate energy-momentum tensor of the matter produced by the collision in the forward light cone. The metric of eq. (3.7) is our formulation of the problem of heavy ion collisions in AdS. Similar metrics were previously considered in modeling heavy ion collisions in AdS_3 in [66] and in AdS_5 in [61, 67].

The metric of eq. (3.7) is illustrated in figure 3. The first two terms in figure 3 (diagrams A and B) correspond to one-graviton exchanges which constitute the individual metrics of each of the nuclei, as shown in eq. (3.1). Our goal below is to calculate the next order correction to these terms, which is shown in the diagram C in figure 3. Higher order graviton exchanges would necessarily involve both nuclei: as the metric (3.1) is an exact solution of Einstein equations, all higher order graviton exchanges with a single nucleus are zero. Indeed, solving Einstein equations order-by-order in the graviton exchanges one could reconstruct any higher order term in the series of figure 3. In the calculations below we will restrict ourselves to diagram C in figure 3, which is the first correction to the sum of the metrics of the two nuclei, and leave calculation of the higher orders for future projects.

4. General perturbative solution

Here we will calculate the diagram C in figure 3 by solving Einstein equations (2.11) perturbatively. Define normalized light-cone components of the nuclear energy-momentum tensors by

$$t_1(x^-) \equiv \frac{2\,\pi^2}{N_c^2}\,\langle T_{1\,--}(x^-)\rangle \tag{4.1}$$

and

$$t_2(x^+) \equiv \frac{2\,\pi^2}{N_c^2}\,\langle T_{2\,++}(x^+)\rangle. \tag{4.2}$$

Using these definitions we rewrite the metric in eq. (3.7) as

$$ds^2 = \frac{L^2}{z^2}\left\{ -2\,dx^+\,dx^- + dx_\perp^2 + dz^2 + t_1(x^-)\,z^4\,dx^{-\,2} + t_2(x^+)\,z^4\,dx^{+\,2} + o(t_1\,t_2)\right\}. \tag{4.3}$$

Notice that there is no higher order corrections containing only powers of $t_1(x^-)$ or of $t_2(x^+)$: they are zero since the single nucleus metric (3.1) and its analogue for nucleus 2 are *exact* solutions of Einstein equations.

We denote by $g_{\mu\nu}$ the metric in AdS$_5$ space dual to heavy ion collisions that we are trying to construct. Then eq. (4.3) can be written as

$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu \tag{4.4}$$

with μ,ν running from 0 to 4. The order-by-order perturbative solution of Einstein equations is obtained by expanding the metric around the empty AdS$_5$ space

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + g_{\mu\nu}^{(1)} + g_{\mu\nu}^{(2)} + \ldots \tag{4.5}$$

where the metric $g_{\mu\nu}^{(n)}$ corresponds to n graviton exchanges. For the energy-momentum tensor of eq. (3.4) the series in eq. (4.5) corresponds to expansion in the powers of μ, while for the energy-momentum tensor of eq. (3.5) the series in eq. (4.5) is an expansion in powers of Λ^2.

Here $g_{\mu\nu}^{(0)}$ is the metric of the empty AdS$_5$ space with non-zero components

$$g_{+-}^{(0)} = g_{-+}^{(0)} = -\frac{L^2}{z^2}, \quad g_{ij}^{(0)} = \delta_{ij}\,\frac{L^2}{z^2}, \quad i,j=1,2, \qquad g_{zz}^{(0)} = \frac{L^2}{z^2}. \tag{4.6}$$

$g_{\mu\nu}^{(1)}$ is the first perturbation of the empty AdS$_5$ space due to the two nuclei

$$g_{--}^{(1)} = t_1(x^-)\,L^2\,z^2, \quad g_{++}^{(1)} = t_2(x^+)\,L^2\,z^2 \tag{4.7}$$

with all the other components zero.

We want to find the next non-trivial correction $g_{\mu\nu}^{(2)}$. By the choice of Fefferman-Graham coordinates one has $g_{z\mu} = g_{\mu z} = 0$ exactly for $\mu \neq z$ and $g_{zz} = L^2/z^2$. Hence the non-trivial components of $g_{\mu\nu}^{(2)}$ are those for $\mu,\nu = 0,\ldots,3$. Due to translational and rotational invariance of the nuclei in the transverse direction $g_{ij}^{(2)} \sim \delta_{ij}$. We thus parametrize the unknown components of $g_{\mu\nu}^{(2)}$ as

$$\begin{aligned} g_{--}^{(2)} &= \frac{L^2}{z^2}\,f(x^+,x^-,z), & g_{++}^{(2)} &= \frac{L^2}{z^2}\,\tilde{f}(x^+,x^-,z), \\ g_{+-}^{(2)} &= -\frac{1}{2}\,\frac{L^2}{z^2}\,g(x^+,x^-,z), & g_{ij}^{(2)} &= \frac{L^2}{z^2}\,h(x^+,x^-,z)\,\delta_{ij} \end{aligned} \tag{4.8}$$

with f, $\tilde{f}$, g and h some unknown functions. Imposing causality we require that functions f, $\tilde{f}$, g and h are zero before the collision, i.e., that before the collision the metric is given only by the empty AdS space and by the contributions of the two nuclei eq. (4.7). Also, according to general properties of $g_{\mu\nu}$ outlined in section 2 (see [81]), we demand that f, $\tilde{f}$, g and h go to zero as z^4 when $z \to 0$.

Using eqs. (4.6), (4.7), and (4.8) in eq. (4.5), plugging the latter into Einstein equations (2.11) and keeping only the terms up to and including the order $g^{(2)}_{\mu\nu}$ we obtain the following set of equations for f, $\tilde{f}$, g and h labeled by the Einstein equations components:

$$(--) \qquad \frac{3}{2\,z}\,f_z - \frac{1}{2}\,f_{z\,z} - h_{x^-\,x^-} = 0, \tag{4.9a}$$

$$(++) \qquad \frac{3}{2\,z}\,\tilde{f}_z - \frac{1}{2}\,\tilde{f}_{z\,z} - h_{x^+\,x^+} = 0, \tag{4.9b}$$

$$\begin{aligned}(+-) \quad & -\frac{5}{4\,z}\,g_z - \frac{1}{z}\,h_z + \frac{1}{4}\,g_{z\,z} - \frac{1}{2}\,g_{x^+\,x^-} \\ & -h_{x^+\,x^-} - \frac{1}{2}\,f_{x^+\,x^+} - \frac{1}{2}\,\tilde{f}_{x^-\,x^-} = 4\,z^6\,t_1(x^-)\,t_2(x^+) - \frac{1}{4}\,z^8\,t_1'(x^-)\,t_2'(x^+),\end{aligned} \tag{4.9c}$$

$$(\perp\perp) \qquad g_z + 5\,h_z - z\,h_{z\,z} + 2\,z\,h_{x^+\,x^-} = 8\,z^7\,t_1(x^-)\,t_2(x^+), \tag{4.9d}$$

$$(zz) \qquad g_z + 2\,h_z - z\,g_{z\,z} - 2\,z\,h_{z\,z} = -32\,z^7\,t_1(x^-)\,t_2(x^+), \tag{4.9e}$$

$$(-z) \qquad -\frac{1}{2}\,f_{x^+\,z} - \frac{1}{4}\,g_{x^-\,z} - h_{x^-\,z} = -z^7\,t_1'(x^-)\,t_2(x^+), \tag{4.9f}$$

$$(+z) \qquad -\frac{1}{2}\,\tilde{f}_{x^-\,z} - \frac{1}{4}\,g_{x^+\,z} - h_{x^+\,z} = -z^7\,t_1(x^-)\,t_2'(x^+). \tag{4.9g}$$

The subscripts z, x^+ and x^- indicate partial derivatives with respect to these variables.

To solve eqs. (4.9) begin by solving eq. (4.9d) for g_z and substituting the result into eq. (4.9e). This gives

$$-3\,h_z + 3\,z\,h_{z\,z} - z^2\,h_{z\,z\,z} + 2\,z^2\,h_{x^+\,x^-\,z} = 16\,z^7\,t_1(x^-)\,t_2(x^+). \tag{4.10}$$

We look for the solution of eq. (4.10) as a series in powers of z^2. Note that $h(x^+, x^-, z)$ goes to zero proportionally to z^4 as $z \to 0$: therefore the series starts at the order z^4 and reads

$$h(x^+, x^-, z) \;=\; z^4 \sum_{n=0}^{\infty} h_n(x^+, x^-)\,z^{2n}. \tag{4.11}$$

Substituting eq. (4.11) into eq. (4.10) and solving it order-by-order in z we can express all the coefficients in the series in terms of the first coefficient $h_0(x^+, x^-)$ and in terms of $t_1(x^-)$ and $t_2(x^+)$ obtaining

$$\begin{aligned} h(x^+, x^-, z) \;&= 4\,z^2\,\frac{I_2(z\sqrt{2\,\partial_+\,\partial_-})}{\partial_+\,\partial_-}\;h_0(x^+, x^-) - 32\,z^2\,\Bigg[I_2(z\sqrt{2\,\partial_+\,\partial_-}) \\ & - \frac{1}{4}\,z^2\,\partial_+\,\partial_- - \frac{1}{24}\,z^4\,(\partial_+\,\partial_-)^2\Bigg]\,\frac{1}{(\partial_+\,\partial_-)^3}\;t_1(x^-)\,t_2(x^+). \end{aligned} \tag{4.12}$$

(Inverse derivatives in eq. (4.12) are canceled by the positive powers of derivatives in the numerators of the appropriate terms.)

As shown in appendix B, plugging eq. (4.12) into eq. (4.9d) one can find $g(x^+, x^-, z)$, and, using eqs. (4.9f) and (4.9g), one can find $f(x^+, x^-, z)$ and $\tilde{f}(x^+, x^-, z)$. Using the obtained expressions to eliminate $f(x^+, x^-, z)$, $\tilde{f}(x^+, x^-, z)$ and $g(x^+, x^-, z)$ in eq. (4.9c) one can see that for the solution of Einstein equations to exist the following condition needs to be satisfied:

$$(\partial_+ \partial_-)^2 h_0(x^+, x^-) = 8\, t_1(x^-)\, t_2(x^+). \tag{4.13}$$

As can be seen from eq. (4.12) the infinite series (4.11) for $h(x^+, x^-, z)$ will then terminate at the order z^6. As shown in appendix B, the solutions for $f(x^+, x^-, z)$, $\tilde{f}(x^+, x^-, z)$ and $g(x^+, x^-, z)$ will also reduce to finite-order polynomials in z^2.

The only other non-vanishing coefficient in the series for $h(x^+, x^-, z)$ in eq. (4.11) is $h_1(x^+, x^-)$ which is related to $h_0(x^+, x^-)$ via

$$h_1(x^+, x^-) = \frac{1}{6}\, \partial_+ \partial_-\, h_0(x^+, x^-). \tag{4.14}$$

Using eq. (4.13) we obtain

$$\partial_+ \partial_-\, h_1(x^+, x^-) = \frac{4}{3}\, t_1(x^-)\, t_2(x^+). \tag{4.15}$$

Eqs. (4.13) and (4.15) allow us to determine the functions h_0 and h_1. Imposing causality by requiring that at time $-\infty$, i.e. long before the collision, the shock waves are unmodified we write

$$h_0(x^+, x^-) = 8 \int\limits_{-\infty}^{x^-} dx'^- \int\limits_{-\infty}^{x'^-} dx''^- \int\limits_{-\infty}^{x^+} dx'^+ \int\limits_{-\infty}^{x'^+} dx''^+\, t_1(x''^-)\, t_2(x''^+) \tag{4.16}$$

and

$$h_1(x^+, x^-) = \frac{4}{3} \int\limits_{-\infty}^{x^-} dx'^- \int\limits_{-\infty}^{x^+} dx'^+\, t_1(x'^-)\, t_2(x'^+). \tag{4.17}$$

In terms of h_0 and h_1 from eqs. (4.16) and (4.17) we write our solution for $h(x^+, x^-, z)$ as

$$h(x^+, x^-, z) = h_0(x^+, x^-)\, z^4 + h_1(x^+, x^-)\, z^6. \tag{4.18}$$

Plugging the solution (4.18) into eq. (4.9d) we solve for $g(x^+, x^-, z)$ to obtain

$$g(x^+, x^-, z) = -2\, h_0(x^+, x^-)\, z^4 - 2\, h_1(x^+, x^-)\, z^6 + \frac{2}{3}\, t_1(x^-)\, t_2(x^+)\, z^8. \tag{4.19}$$

Substituting the solutions for h and g from eqs. (4.18) and (4.19) into eq. (4.9f) we solve for $f(x^+, x^-, z)$ to find

$$f(x^+, x^-, z) = -\lambda_1(x^+, x^-)\, z^4 - \frac{1}{6}\, \partial_-^2 h_0(x^+, x^-)\, z^6 - \frac{1}{16}\, \partial_-^2 h_1(x^+, x^-)\, z^8 \tag{4.20}$$

with $\lambda_1(x^+, x^-)$ given by

$$\lambda_1(x^+, x^-) \;=\; \int\limits_{-\infty}^{x^+} dx'^+ \,\partial_- h_0(x'^+, x^-). \tag{4.21}$$

Similarly eq. (4.9g) yields

$$\tilde{f}(x^+, x^-, z) \;=\; -\lambda_2(x^+, x^-)\, z^4 - \frac{1}{6}\,\partial_+^2 h_0(x^+, x^-)\, z^6 - \frac{1}{16}\,\partial_+^2 h_1(x^+, x^-)\, z^8 \tag{4.22}$$

with

$$\lambda_2(x^+, x^-) \;=\; \int\limits_{-\infty}^{x^-} dx'^- \,\partial_+ h_0(x^+, x'^-). \tag{4.23}$$

Eqs. (4.18), (4.19), (4.20) and (4.22) provide us with the solution of eq. (4.9) with the causal initial condition requiring all these functions to go to zero at infinitely early times.

Using eq. (2.3) one can obtain the contribution of $g^{(2)}_{\mu\nu}$ to the expectation value of the energy-momentum tensor at the boundary of the AdS space from eqs. (4.18), (4.19), (4.20) and (4.22) and eq. (4.8):

$$\begin{aligned}
\langle T_{--}\rangle &= -\frac{N_c^2}{2\,\pi^2}\,\lambda_1(x^+, x^-), & \langle T_{++}\rangle &= -\frac{N_c^2}{2\,\pi^2}\,\lambda_2(x^-, x^+), \\
\langle T_{+-}\rangle &= \frac{N_c^2}{2\,\pi^2}\,h_0(x^-, x^+), & \langle T_{ij}\rangle &= \frac{N_c^2}{2\,\pi^2}\,\delta_{ij}\,h_0(x^-, x^+).
\end{aligned} \tag{4.24}$$

Given $t_1(x^-)$ and $t_2(x^+)$, one can use eqs. (4.16), (4.21) and (4.23) to find h_0, λ_1 and λ_2, and then use eq. (4.24) to construct the energy-momentum tensor of the gauge theory.

5. Physical shock waves: nuclear stopping

To understand our solution given by eqs. (4.18), (4.19), (4.20) and (4.22) let us consider a specific example of shock waves with the boundary energy-momentum tensor given by eq. (3.3). To be able to better understand physical properties of the solution, let us "smear" the shock waves over some finite longitudinal distance. If one imagines shock waves representing a large nucleus, such nucleus moving in the x^+-direction in a boosted ultrarelativistic frame would have a longitudinal extent

$$a \,\propto\, R\,\frac{\Lambda}{p^+} \,\propto\, \frac{A^{1/3}}{p^+} \tag{5.1}$$

with R the nuclear radius, Λ the typical transverse momentum scale in the problem ($R \propto A^{1/3}/\Lambda$ with A the atomic number), and p^+ the large longitudinal momentum of the nucleus. (Here Λ/p^+ is the boost factor.)

Assuming that the nucleus has equal thickness a at all impact parameters, we replace the delta-function in eq. (3.3) with two theta-functions to write

$$t_1(x^-) \;=\; 2\,\pi^2\,\frac{\mu}{a}\,\theta(x^-)\,\theta(a - x^-) \tag{5.2}$$

for the first nucleus and

$$t_2(x^+) \; = \; 2\,\pi^2\,\frac{\mu}{a}\,\theta(x^+)\,\theta(a - x^+) \tag{5.3}$$

for the second one. For simplicity we assumed that the nuclei are identical and are scattering with equal momenta $p_1^+ = p_2^-$, such that $\mu = \mu_1 = \mu_2$ and $a = a_1 = a_2$.

Plugging eqs. (5.2) and (5.3) into eq. (4.16) we immediately obtain

$$\begin{aligned} h_0(x^+, x^-) \; =& 8\,\frac{\mu^2}{a^2}\,(2\,\pi^2)^2 \left[\theta(x^-)\,\theta(a - x^-)\,\frac{x^{-\,2}}{2} + \theta(x^- - a)\,a\,\left(x^- - \frac{a}{2}\right) \right] \\ & \left[\theta(x^+)\,\theta(a - x^+)\,\frac{x^{+\,2}}{2} + \theta(x^+ - a)\,a\,\left(x^+ - \frac{a}{2}\right) \right]. \end{aligned} \tag{5.4}$$

eq. (4.21) then gives

$$\begin{aligned} \lambda_1(x^+, x^-) \; =& 8\,\frac{\mu^2}{a^2}\,(2\,\pi^2)^2 \left[\theta(x^-)\,\theta(a - x^-)\,x^- + \theta(x^- - a)\,a \right] \\ & \left[\theta(x^+)\,\theta(a - x^+)\,\frac{x^{+\,3}}{6} + \theta(x^+ - a)\,a\,\left(\frac{a^2}{6} + \frac{x^{+\,2}}{2} - \frac{a\,x^+}{2}\right) \right], \end{aligned} \tag{5.5}$$

while eq. (4.23) due to the fact that nuclei are identical leads to

$$\lambda_2(x^+, x^-) \; = \; \lambda_1(x^-, x^+). \tag{5.6}$$

Eqs. (5.4), (5.5) and (5.6), along with eq. (4.24), give us the order μ^2 energy-momentum tensor. Let us study its properties. First of all, away from the light cone for $x^+, x^- \gg a$ (or in the limit of infinitely thin nuclei, which can be recovered by taking $a \to 0$) one has

$$\begin{aligned} & h_0(x^+, x^-)\Big|_{x^+, x^- \gg a} \approx 8\,(2\,\pi^2)^2\,\mu^2\,x^+\,x^-, \qquad \lambda_1(x^+, x^-)\Big|_{x^+, x^- \gg a} \approx 8\,(2\,\pi^2)^2\,\mu^2\,\frac{x^{+\,2}}{2}, \\ & \lambda_2(x^+, x^-)\Big|_{x^+, x^- \gg a} \approx 8\,(2\,\pi^2)^2\,\mu^2\,\frac{x^{-\,2}}{2}. \end{aligned} \tag{5.7}$$

Substituting eq. (5.7) into eq. (4.24) one gets for the forward light-cone far away from the nuclei

$$\begin{aligned} \langle T_{--} \rangle \; &= \; -\,8\,\pi^2\,N_c^2\,\mu^2\,x^{+\,2}, & \langle T_{++} \rangle \; &= \; -\,8\,\pi^2\,N_c^2\,\mu^2\,x^{-\,2}, \\ \langle T_{+-} \rangle \; &= 8\,\pi^2\,N_c^2\,\mu^2\,\tau^2, & \langle T_{i\,j} \rangle \; &= 8\,\pi^2\,\delta_{i\,j}\,N_c^2\,\mu^2\,\tau^2. \end{aligned} \tag{5.8}$$

This is the same result as obtained in [67]. The energy-momentum tensor in eq. (5.8) is rapidity-independent, as its components contain no rapidity dependence apart from the trivial factors needed for Lorentz-properties of the tensor.

For two colliding nuclei of infinite transverse extent (the Bjorken case [70]) the most general parameterization of the rapidity-independent energy-momentum tensor is [43]

$$\begin{aligned} T_{--} &= [\epsilon(\tau) + p_3(\tau)] \left(\frac{x^+}{\tau}\right)^2, \\ T_{++} &= [\epsilon(\tau) + p_3(\tau)] \left(\frac{x^-}{\tau}\right)^2, \\ T_{+-} &= [\epsilon(\tau) - p_3(\tau)] \, \frac{1}{2}, \\ T_{ij} &= \delta_{ij} \, p(\tau), \end{aligned} \tag{5.9}$$

where $\epsilon(\tau)$, $p(\tau)$ and $p_3(\tau)$ are the energy density, transverse pressure and longitudinal pressure components of the energy-momentum tensor at mid-rapidity ($x^3 = 0$). At $x^3 = 0$ the tensor (5.9) looks like

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & p(\tau) & 0 & 0 \\ 0 & 0 & p(\tau) & 0 \\ 0 & 0 & 0 & p_3(\tau) \end{pmatrix}. \tag{5.10}$$

One can easily show that conservation of energy and momentum condition

$$\partial_\mu T^{\mu\nu} \, = \, 0 \tag{5.11}$$

applied to the tensor (5.9) gives

$$\frac{d\epsilon}{d\tau} \, = \, -\frac{\epsilon + p_3}{\tau}. \tag{5.12}$$

The condition $\partial_\mu T^{\mu\nu} = 0$ follows from Einstein equations if one uses gauge-gravity duality to obtain the energy momentum tensor. For conformal field theories the energy-momentum tensor is traceless

$$T_\mu^\mu \, = \, 0, \tag{5.13}$$

which implies

$$\epsilon \, = \, 2\, p + p_3. \tag{5.14}$$

Eqs. (5.12) and (5.14) relate $\epsilon(\tau)$, $p(\tau)$ and $p_3(\tau)$ to each other, such that knowing one of these functions is sufficient to reconstruct the others.

Comparing eq. (5.8) with eq. (5.9) we read off the energy density at mid-rapidity

$$\epsilon(\tau) \, = \, 4\, \pi^2 \, N_c^2 \, \mu^2 \, \tau^2. \tag{5.15}$$

Such energy density at mid-rapidity is problematic. It grows with the proper time τ. One can show that the requirement that the energy density of produced matter is positive-definite in any frame in particular demands that [54]

$$\epsilon'(\tau) \, \leq \, 0. \tag{5.16}$$

eq. (5.15) obviously violates the condition (5.16): this means our solution gives *negative energy density* in some frames. Such result is clearly unphysical.

It is important to understand the origin of this negativity of the energy density. First we note that, as one can easily see, the energy density becomes negative in the frames with the time direction being close to the light cones (the shock waves). To investigate the region around the shock waves further, let us concentrate on the shock wave corresponding to the nucleus 1 after the collision. Let us study what happens to, say, the middle of the nucleus, which is located at $x^- = a/2$, after the collision. The important component of the energy-momentum tensor is $\langle T_{--}\rangle$, since it contains the (large) momentum component of the nucleus along its light cone. Using eq. (4.24) along with eq. (5.5) at $x^- = a/2$ yields for $x^+ \gg a$ (after the collision)

$$\langle T_{--}(x^+ \gg a, x^- = a/2)\rangle \;=\; N_c^2\,\frac{\mu}{a} - 4\,\pi^2\,N_c^2\,\mu^2\,x^{+\,2}, \tag{5.17}$$

where the first term on the right is due to the original shock wave obtained by using eqs. (5.2) and (4.1) at $x^- = a/2$.

eq. (5.17) shows that $\langle T_{--}\rangle$ of a nucleus becomes *negative* at light-cone times

$$x^+ \;\sim\; \frac{1}{\sqrt{\mu\,a}}. \tag{5.18}$$

Indeed zero $\langle T_{--}\rangle$ would mean a complete *stopping* of the shock wave and the corresponding nucleus. We therefore conclude that negativity of energy density (5.15) in fact is a signal of complete stopping of the colliding nuclei after the collision!

Indeed at times $x^+ \sim 1/\sqrt{\mu\,a}$ higher order corrections to the metric due to higher graviton exchanges would become important preventing $\langle T_{--}\rangle$ from going negative. Nevertheless, eq. (5.17) demonstrates that at rather short times $x^+ \sim 1/\sqrt{\mu\,a}$ the nucleus looses the amount of energy comparable to its initial incoming energy, and thus is likely to stop.

One should also point out that eq. (5.17) gives $\langle T_{--}\rangle$ of the center of the nucleus ($x^- = a/2$): other slices of the nucleus located at different x^- would also stop, but at slightly different times x^+. All stopping would happen at the same parametric time given by eq. (5.18).

To better understand the stopping time we use eqs. (3.4) and (5.1) to re-write eq. (5.18) as

$$x^+ \;\sim\; \frac{1}{\Lambda\,A^{1/3}}. \tag{5.19}$$

The stopping time appears to be energy-independent! It is given by the inverse of the typical transverse momentum scale Λ in the problem. It also decreases with the increasing size of the nucleus A.

Let us pause to interpret the main result of this section. It appears that two colliding ultrarelativistic shock waves would come to a complete stop shortly after the collision. One can understand this in terms of creation of a black hole: both shock waves carry large energy, which functions as a mass. There is a horizon radius in AdS space corresponding

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to that mass/energy. For nuclei of infinite transverse extent under consideration the shock waves always come closer to each other than the horizon radius corresponding to the energy they carry. A black hole is then formed and the shock waves stop completely within the black hole's horizon radius. The picture is similar to black hole production in collisions at transplanckian energies, which has been recently discussed in the literature [72 – 74].

One could picture a collision of two nuclei of finite transverse extent. If the impact parameter of such a collision is larger than the horizon radius, no black hole will be formed and the nuclei will not stop. However, in such case there will probably be no thermal matter produced in the boundary theory either.

It is interesting to note that the stopping time (5.19) is independent of energy: indeed, on one hand if one increases the momentum of the shock wave it is harder to stop it. On the other hand, increasing the energy of the shock waves tends to reduce the radius of the horizon, trying to make the shock waves stop faster. We interpret the result of eq. (5.19) as the cancellation of the two effects, leading to energy-independence of the stopping time.

If the nuclei stop completely in the collision, the strong interactions between them are almost certain to thermalize the system. Indeed if the interactions were strong enough to stop the nuclei, they should be strong enough to thermalize the resulting medium. The dynamics of such a rotationally-invariant thermal medium was originally described hydrodynamically by Landau in [75] and is commonly referred to as Landau hydrodynamics. Hence our conclusion is that modeling a collision of two nuclei by two physical colliding shock waves in AdS necessarily leads to complete nuclear stopping, and probably to thermalization of the system and the subsequent dynamics describable by Landau hydrodynamics.

The fact that nuclei do not stop instantaneously, but require certain (short) time (5.19) to stop avoids the standard counter-argument [89] against Landau hydrodynamics [75], which suggests that it would violate the uncertainty principle if the stopping was instantaneous. Hence the picture is intrinsically consistent.

One may wonder whether our result of eq. (5.17) is specific to the shape of the shock waves we have considered in eqs. (5.2) and (5.3). In fact our conclusion of complete stopping is valid in general: as one can see from eq. (4.16), any non-negative energy-momentum tensor of the shock wave, which is positive in a localized region of x^- (x^+) axis, would give $h_0 \sim x^+ x^-$ at late times. Eqs. (4.24) and (5.9) would then give

$$\epsilon(\tau) \sim p(\tau) \sim -p_3(\tau) \sim \tau^2, \tag{5.20}$$

just like in our case considered above. Hence the energy density of such system would never be non-negative in all frames, signaling the stopping of shock waves. Finally, eq. (4.21) would give $\lambda_1 \sim x^{+\,2}$, such that the correction to the energy-momentum tensor on the light cone would again be

$$\langle T_{--} \rangle \sim -x^{+\,2} \tag{5.21}$$

indicating that at some large enough light-cone time x^+ the nucleus would run out of its momentum. This proves the shock waves stopping independent of the shape of the shock waves profiles.

Let us close this section by pointing out that the result in eq. (5.15) can be easily obtained (at the parametric level) for infinitely thin nuclei by noticing that the diagram in figure 3C, which contributes to the metric giving the energy density in eq. (5.15), is of the order of μ^2. Hence the contribution to ϵ at this order should be of the order of μ^2. However, energy density has dimension of mass to the fourth power, while μ^2 has dimensions of mass to the sixth power. To make the dimensions right we use the only other dimensionful quantity in the boundary gauge theory in the forward light cone: the proper time τ. This gives $\epsilon \sim \mu^2\tau^2$, in agreement with eq. (5.15). We noted above that expansion in graviton exchanges in the bulk is equivalent to expansion in the powers of μ for the energy-momentum tensor of the shock waves in eq. (3.3). The only way to make a dimensionless expansion parameter from μ in the boundary theory is to multiply it by τ^3. Now we see that for energy density the expansion parameter is in fact $\mu\,\tau^3$, which has been noticed in [67] before. However, now we understand that each power of this expansion parameter corresponds to a graviton exchange between the boundary and the bulk.

6. Unphysical shock waves: energy density of the produced medium

In the above section we came to the conclusion that at very strong coupling colliding nuclei completely stop in a collision (for central collisions), forming a medium described by Landau hydrodynamics. However, due to asymptotic freedom of QCD, we know that small-coupling effects play an important role in heavy ion collisions and in high energy collisions in general. The success of Color Glass Condensate [39 − 41] based models in describing RHIC data (see e.g. [90] and references therein) suggests that weakly coupled effects are present in the actual heavy ion collisions at RHIC, at least at very early times during and after the collision. While a comprehensive description of both weakly-coupled initial dynamics and strongly-coupled dynamics of the produced medium is not feasible at this point, here we will suggest a model capturing some of the feature of the weakly coupled collisions.

We begin by noting that, in the weak coupling limit, the colliding nuclei do not stop, as we observed in the previous section for the strong coupling case. Instead the valence quarks and other large Bjorken-x (hard) partons are usually assumed to go through each other without deflection, shedding off the softer (small-x) virtual partons, which are left behind and quickly go on mass shell, i.e., become real [77, 26, 27, 78, 79, 30, 29, 42, 31, 44]. The medium made out of these small-x partons after the collision has a non-negative energy density in any frame [32, 68, 42, 79, 30]. We will then proceed by requiring that the energy density of the produced strongly coupled medium should also be non-negative.

We want to model the heavy ion collisions by colliding two shock waves. The conclusion of the previous section was that any localized non-negative $\langle T_{1\,--}\rangle$ of a shock wave, such that

$$\int\limits_{-\infty}^{\infty} dx^- \langle T_{1\,--}(x^-)\rangle \, > \, 0 \tag{6.1}$$

(with an analogous condition imposed on the $\langle T_{2\,++}(x^+)\rangle$ component of the energy momentum tensor of the other shock wave) leads to the energy density scaling of eq. (5.20). This violates the condition (5.16) derived in [54] and results in the negative energy density of the produced matter in some frames. The only way around such an unphysical conclusion appears to be to require that

$$\int_{-\infty}^{\infty} dx^- \langle T_{1\,--}(x^-)\rangle \;=\; 0, \qquad \int_{-\infty}^{\infty} dx^+ \langle T_{2\,++}(x^+)\rangle \;=\; 0. \tag{6.2}$$

Indeed the conditions (6.2) can only be satisfied in a physical world if there is no shock waves, in which case their energy would be zero. Such a trivial scenario is not what we have in mind.

Instead, we propose using unphysical not positive-definite quantities for $\langle T_{1\,--}(x^-)\rangle$ and
$\langle T_{2\,++}(x^+)\rangle$, which integrate out to zero satisfying eq. (6.2). Indeed such objects would be completely unphysical, as they would contain regions of negative energy density. They can not be obtained from an underlying string theory either. However, we intend to use them in gauge-gravity duality only. On both sides of the gauge-gravity duality our non-positive energy momentum tensors should be regarded as external sources to the theory. The conclusion we have reached is that to have non-negative energy density in the forward light cone one needs unphysical negative energy shock waves on the light cone itself.

Indeed our proposal of zero-energy shock waves may not be a unique way of modeling weak coupling effects in heavy ion collisions in the AdS/CFT framework. One may also try to construct a metric with the CGC-inspired energy-momentum tensor for the gauge theory at early proper time and evolve it in time using Einstein equations. However, constructing a metric which is a valid initial condition for Einstein equations at early times and accounts for perturbative features of the collision appears to be difficult. If one insists on modeling the heavy ion collisions by two colliding shock waves, our zero-energy shock wave proposal is the only way to mimic the weak coupling effects at initial stages of the collision.

Inspired by eqs. (3.3) and (3.5), which contain two factors of transverse momenta times some function of longitudinal coordinates and momenta, we suggest describing the energy-momentum tensors of the colliding nuclei by

$$\begin{aligned} \langle T_{1\,--}(x^-)\rangle &= \frac{N_c^2}{2\,\pi^2}\,\Lambda_1^2\,\delta'(x^-) \\ \langle T_{2\,++}(x^+)\rangle &= \frac{N_c^2}{2\,\pi^2}\,\Lambda_2^2\,\delta'(x^+) \end{aligned} \tag{6.3}$$

corresponding to

$$\begin{aligned} t_1(x^-) &= \Lambda_1^2\,\delta'(x^-) \\ t_2(x^+) &= \Lambda_2^2\,\delta'(x^+) \end{aligned} \tag{6.4}$$

in the shock waves metric in eq. (4.3). $\delta'(x)$ denotes the derivative of a delta-function. Clearly the energy-momentum tensors in eq. (6.3) satisfy eq. (6.2). What we loose in this description is the relation between the transverse momentum scales Λ_1^2 and Λ_2^2 describing the two nuclei in eq. (6.3) and the actual physical parameters describing the real nuclei, since our energy-momentum tensors in eq. (6.3) are not physical and we can not relate them to the energy-momentum tensors of the two nuclei.

Before we perform any calculations we can already guess the answer using the dimensional analysis outlined in section 5. This time each vertex in figure 3C brings in a factor of Λ_1^2 and Λ_2^2, such that the diagram is proportional to $\Lambda_1^2\,\Lambda_2^2$. Hence the resulting energy density of the boundary theory is proportional to $\epsilon \sim \Lambda_1^2\,\Lambda_2^2$. Since the dimensions of ϵ and $\Lambda_1^2\,\Lambda_2^2$ match, no powers of τ are needed this time. Hence we conclude that the energy density of the matter produced by the two shock waves (6.4) at the lowest order in graviton exchanges is $\epsilon \sim \Lambda_1^2\,\Lambda_2^2$, i.e. a *constant* of time, as was suggested in [1]. $\epsilon \sim \Lambda_1^2\,\Lambda_2^2$ immediately satisfies the condition (5.16) derived in [54]: hence the energy density is non-negative in any reference frame. Finally, now the graviton exchanges between the boundary and the bulk should correspond to powers of $\Lambda_1^2\,\tau^2$ or $\Lambda_2^2\,\tau^2$ in the gauge theory. Thus the early-time expansion for the energy density should contain powers of $\Lambda_1^2\,\tau^2$ and $\Lambda_2^2\,\tau^2$. Therefore, while we do not know how to relate Λ_1^2 and Λ_2^2 to the physical observables, we still can systematically construct the dual geometry to the collision by expanding the metric in the powers of $\Lambda_1^2\,\tau^2$ and $\Lambda_2^2\,\tau^2$, and hopefully would be able to arrive at the thermalization/isotropization transition in the Bjorken sense [70, 43].

The actual calculations are performed easily. Plugging eq. (6.4) into eq. (4.16) yields

$$h_0(x^+, x^-) \;=\; 8\,\Lambda_1^2\,\Lambda_2^2\,\theta(x^-)\,\theta(x^+). \tag{6.5}$$

Using eqs. (4.17), (4.18), (4.19), (4.20), (4.21), (4.22), (4.23), and (4.8) we find the second order correction to the metric (4.3)

$$\begin{aligned}
g_{--}^{(2)} &= \frac{L^2}{z^2}\,\Lambda_1^2\,\Lambda_2^2\,\left[-8\,\delta(x^-)\,x^+\,\theta(x^+)\,z^4 - \frac{4}{3}\,\delta'(x^-)\,\theta(x^+)\,z^6 - \frac{1}{12}\,\delta''(x^-)\,\delta(x^+)\,z^8\right],\\
g_{++}^{(2)} &= \frac{L^2}{z^2}\,\Lambda_1^2\,\Lambda_2^2\,\left[-8\,x^-\,\theta(x^-)\,\delta(x^+)\,z^4 - \frac{4}{3}\,\theta(x^-)\,\delta'(x^+)\,z^6 - \frac{1}{12}\,\delta(x^-)\,\delta''(x^+)\,z^8\right],\\
g_{+-}^{(2)} &= -\frac{1}{2}\,\frac{L^2}{z^2}\,\Lambda_1^2\,\Lambda_2^2\,\left[-16\,\theta(x^-)\,\theta(x^+)\,z^4 - \frac{8}{3}\,\delta(x^-)\,\delta(x^+)\,z^6 + \frac{2}{3}\,\delta'(x^-)\,\delta'(x^+)\,z^8\right],\\
g_{ij}^{(2)} &= \frac{L^2}{z^2}\,\delta_{ij}\,\Lambda_1^2\,\Lambda_2^2\,\left[8\,\theta(x^-)\,\theta(x^+)\,z^4 + \frac{4}{3}\,\delta(x^-)\,\delta(x^+)\,z^6\right],
\end{aligned} \tag{6.6}$$

where the double prime denotes the second derivative.

One might be concerned with the fact that order-z^4 components of $g_{--}^{(2)}$ and $g_{++}^{(2)}$ contain a negative contributions localized to the light cone and growing with time. They may be interpreted, just like eq. (5.17) above, as a signal of stopping of the shock waves. However, our shock waves start out carrying negative energy and momentum densities. Hence the concept of stopping is ill-defined for our shock waves, because they themselves are ill-defined as physical objects, and are only used as some sources providing us with realistic dynamics of the produced medium in the forward light cone.

Using eqs. (4.24) and (5.9) along with eq. (2.3) we can read off the energy density and the pressure components in the forward light cone from eq. (6.6)

$$\begin{aligned}\epsilon(\tau) &= \frac{N_c^2}{\pi^2} 4\,\Lambda_1^2\,\Lambda_2^2, \\ p(\tau) &= \frac{N_c^2}{\pi^2} 4\,\Lambda_1^2\,\Lambda_2^2, \\ p_3(\tau) &= -\frac{N_c^2}{\pi^2} 4\,\Lambda_1^2\,\Lambda_2^2. \end{aligned} \tag{6.7}$$

Once again, just like in CGC [32, 68] and as was obtained in [1], the strongly-coupled medium starts out very anisotropic, with a negative longitudinal pressure. Eqs. (6.6), combined with the lower order metric (4.3) and eqs. (6.4), allow for a systematic construction of the metric as an expansion in graviton exchanges, to be performed elsewhere [91].

Negativity of the longitudinal pressure p_3 in eq. (6.7) is intimately connected with energy conservation. One can easily see from eq. (5.12) that if the energy density scales as $\epsilon \sim 1/\tau$ then $p_3 = 0$. For the energy density which falls off slower with τ, using eq. (5.12) one gets negative p_3, in agreement with eq. (6.7). For the energy density falling off with τ faster than $1/\tau$ one would get positive p_3: however, such behavior of energy density at early time would violate energy conservation. The net energy of the produced medium at early times is proportional to $E \sim \epsilon\,\tau$. The energy density which increases faster than $1/\tau$ at small τ would then lead to infinite energy of the produced medium at very early times, violating energy conservation. Hence energy density at early times can not scale faster than $1/\tau$, leading to negative or zero longitudinal pressure p_3.

7. The dilaton

Let us briefly touch upon one related topic which may become important at higher order in graviton exchanges. Immediately after the collision of two heavy ions the medium is not equilibrated yet. The magnitudes squared for the chromo-electric and chromo-magnetic fields are not equal to each other. This means that the expectation value of the gluonic field strength squared should not be zero. In fact, at weak coupling a CGC calculation at the lowest non-trivial order (order α_s^3) performed along the lines of [43, 27] yields

$$\langle \mathrm{tr} F_{\mu\nu}^2 \rangle \;=\; -4\,\alpha_s^3\,C_F\,\frac{A^2}{S_\perp^2}\int \frac{d^2k_T}{k_T^2}\,\left[J_0^2(k_T\tau) - J_1^2(k_T\tau)\right]. \tag{7.1}$$

Here A and $S_\perp$ are the atomic number and the transverse area of the two identical colliding nuclei and $C_F = (N_c^2-1)/2N_c$. To obtain eq. (7.1) one should substitute the gluon field from eq. (12) in [27] into the Abelian part of $\mathrm{tr}F_{\mu\nu}^2$ and average the resulting expression in the wave functions of both nuclei (see also [92, 43]). At early times eq. (7.1) gives

$$\langle \mathrm{tr} F_{\mu\nu}^2 \rangle\Big|_{Q_s\tau\ll 1} \;\approx\; -4\,\pi\,\alpha_s^3\,C_F\,\frac{A^2}{S_\perp^2}\,\ln\frac{1}{Q_s^2\,\tau^2} \tag{7.2}$$

where $Q_s^2 = 4\,\pi\,\alpha_s^2\,A/S_\perp$ is the saturation scale of the nuclei which regulates the infrared divergence in eq. (7.1) when higher order rescatterings are included. Similar to [32, 68] one may conclude that the scaling (7.2) is true for the all-order classical gluon field [77, 26, 27, 78, 79, 30, 29, 42, 31]. We get

$$\langle \mathrm{tr} F_{\mu\nu}^2 \rangle \Big|_{Q_s \tau \ll 1} \propto -\frac{Q_s^4}{\alpha_s} \ln \frac{1}{Q_s^2 \tau^2}. \tag{7.3}$$

As $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle = 2(B^2 - E^2)$ with B and E the chromo-magnetic and chromo-electric fields, we conclude that at weak coupling the medium at the early stages of the collisions is dominated by chromo-electric fields (see also [93]).

To introduce non-zero expectation value for $\mathrm{tr} F_{\mu\nu}^2$ in AdS one needs to include the dilaton field φ, as [82, 83, 71]

$$\frac{1}{4 g_{YM}^2} \langle \mathrm{tr} F_{\mu\nu}^2 \rangle = \frac{N_c^2}{2\pi^2} \lim_{z \to 0} \frac{\varphi(x^+, x^-, z)}{z^4}. \tag{7.4}$$

The dilaton couples to the metric through modified Einstein equations (2.13) and through the Klein-Gordon equation (2.14).

The metrics of the incoming shock waves like (3.1) are solutions of Einstein equations (2.11) with zero dilaton fields. This implies that the dilaton field is zero for a single nucleus and, using eq. (7.4), $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle = 0$ for a single nucleus as well. This agrees with the fact that at weak coupling $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle = 0$ too, as the electric and magnetic fields of a single ultrarelativistic charge are equal to each other (the equivalent photon/gluon approximation) [24, 84].

At strong coupling the dilaton field may become non-zero after the collision. As one can see from [71], an investigation of late-time dynamics of the strongly-coupled medium requires a dilaton field leading to non-zero $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle$. Since the dilaton field of each of the shock waves is zero, we may only expect that the dilaton field produced in the collision would depend on energy-momentum tensors of both shock waves. Therefore, using the expansion in Λ_1^2 and Λ_2^2 of section 6, one may expect that at the lowest non-trivial order

$$\langle \mathrm{tr} F_{\mu\nu}^2 \rangle \sim \Lambda_1^2 \Lambda_2^2 \tag{7.5}$$

corresponding to the dilaton field (see eq. (7.4))

$$\varphi(x^+, x^-, z) \sim \Lambda_1^2 \Lambda_2^2 z^4. \tag{7.6}$$

However, the sign of the dilaton field and $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle$ can not be determined from these dimensional considerations. (In fact eqs. (2.13) and (2.14) are invariant under $\varphi \to -\varphi$ and do not fix the sign of the dilaton field.) Instead of the logarithmic divergence (7.3) of the perturbation theory, we anticipate the expectation value of $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle$ to go to a constant at early times.

The dilaton field from eq. (7.6) would affect the metric only at the order $\Lambda_1^4 \Lambda_2^4$, as can be seen from eq. (2.13), and therefore we could safely neglect it in the above discussion and in [1]. It would only enter eq. (2.13) at the same order as the four-graviton exchanges.

We stress that while we do not know whether non-zero dilaton field would arise at higher orders in our expansion in graviton exchanges, if it does come in we expect it to be of the form shown in eq. (7.6) (at the lowest order) leading to a constant $\langle \mathrm{tr} F_{\mu\nu}^2 \rangle$ at early times shown in eq. (7.5).

8. Conclusions

Let us summarize the main points of our paper. In section 5 we have demonstrated that a collision of real and physical shock waves in AdS would lead to a full stopping of the shock waves. It is likely that a black hole would be created in AdS space immediately afterwards. For the gauge theory this implies that two nuclei colliding at very strong coupling would stop almost immediately after the collision, after a time interval of the order of $t_{stop} \sim 1/\Lambda$ with Λ some typical transverse momentum of the problem. Thus for RHIC and LHC one might expect $t_{stop} \approx 1\,\text{fm}$. After the stop, creation of the black hole in the bulk likely translates into thermalization and Landau hydrodynamics description of the dynamics for the created medium. This is a possible scenario advocated in [76].

However, we consider it to be more likely that the physics of the initial stages of heavy ion collisions is weakly coupled. This point of view is supported by the many successes of CGC approaches to heavy ion collisions [33, 34]. In the weak coupling scenario the hard (large-x) parts of the nuclear wave functions simply go through each other in the collisions without deflection or recoil. At the same time the produced thermalized medium should still be strongly-coupled. While no comprehensive single description of both the weakly-coupled early stages and the strongly-coupled medium in the final state exists, in section 6 we constructed a model which appears to capture the main features of the weakly-coupled initial state by allowing the energy density to be negative (only) on the light cone. The corresponding shock waves carry both positive and negative energy density and are thus unphysical. They need to be thought about as some external sources for the gravitational field used in gauge-gravity duality. The energy density of the produced medium in the forward light-cone is non-negative: in fact we recovered our earlier result of [1] that the energy density starts out as a constant at early proper times.

We have thus arrived at the following conclusion. If the coupling constant in heavy ion collisions is large throughout the collision this would lead to nuclear stopping followed by Landau hydrodynamics. In the more realistic scenario, according to present phenomenological evidence, in which both the nuclear wave functions and the primary particle production are weakly-coupled, Bjorken hydrodynamics could be still achieved if the coupling constant quickly becomes large. Indeed, as shown in [54], a purely strong coupling approach to the study of late-time dynamics leads to Bjorken hydrodynamics. However, as we argued in the paper, Bjorken hydrodynamics can not result from strong coupling dynamics only.

When this paper was in the final stages of preparation, a preprint [94] was posted on the arXiv, where a similar conclusion about stopping of shock waves has been reached. Also, very recently a new version of [67] appeared on the arXiv, where the possibility of the shock wave energy-momentum tensor as in our eq. (6.3) was briefly mentioned.

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A. A model for the energy-momentum tensor of an ultra-relativistic nucleus

We begin by considering the classical electromagnetic potential of a point-like particle of charge g moving at speed v along the positive z direction. In the covariant gauge it is given by

$$A_\pm = \frac{g}{4\,\pi}\frac{(1\mp v)/\sqrt{2}}{\left[\frac{1}{2}\left((1+v)\,x^- - (1-v)\,x^+\right)^2 + (1-v^2)\,x_\perp^2\right]^{1/2}}\,, \quad A_i = 0 \;\; (i=1,2). \tag{A.1}$$

The energy-momentum tensor associated to this field is

$$T_{\mu\nu} = -F_{\mu\rho}\,F_\nu{}^\rho + \frac{1}{4}\,\eta_{\mu\nu}\,F_{\rho\,\sigma}F^{\rho\,\sigma}\,, \tag{A.2}$$

where η is the Minkowski metric in four dimensions. In the limit $v \to 1$, its only non-vanishing component is

$$T_{--} = (\partial_i\,A_-)^2 = \frac{\alpha}{2\pi}\frac{(1-v^2)^2\,x_\perp^2}{\left[2x^{-2} + (1-v^2)x_\perp^2\right]^3}\,, \tag{A.3}$$

where $\alpha = g^2/4\pi$. In the strict limit $v=1$, T_{--} is singular at $x^-=0$. To clarify the nature of this singularity we consider the following integral:

$$\int_{-\infty}^{\infty} dx\,\frac{\epsilon^4}{(x^2+\epsilon^2)^3}\,f(x) \;=(2\pi\,i)\,\epsilon^4\,\frac{1}{2!}\,\frac{d^2}{d\,z^2}\,\frac{f(z)}{(z+i\,|\epsilon|)^3}\bigg|_{z=i\,|\epsilon|} \overset{\epsilon\to 0}{=} \frac{3\,\pi}{8}\,\frac{f(0)}{|\epsilon|}, \tag{A.4}$$

where $f(x)$ is an arbitrary analytic function that falls off at infinity rapidly enough for the previous integral to be well defined. From eq. (A.3) and eq. (A.4) we get

$$T_{--} = \frac{\alpha}{2\pi}\frac{8}{3\pi}\,\frac{\delta(x^-)}{|x^-|}\,\frac{1}{x_\perp^2} \tag{A.5}$$

The previous equation serves as a starting point to build up a model for the energy-momentum tensor of an ultra-relativistic nucleus, T_{--}^{nucl}. We envisage the nucleus as consisting of A *nucleons*, each of them containing N_c^2 *valence* gluons. Assuming an uniform distribution of nucleons inside the nucleus, averaging over transverse position and summing over all nucleons, we write

$$\langle T_{--}^{nucl}\rangle = N_c^2\,\frac{A}{S_\perp}\int d^2x_\perp\,T_{--} \;=\frac{N_c^2\,\alpha\,N_c}{\pi}\,\frac{\delta(x^-)}{|x^-|}\,\frac{8}{3}\,\frac{A}{S_\perp}\,\ln\left(\frac{R}{\rho}\right), \tag{A.6}$$

where $S_\perp$ is the nuclear transverse area, R is the nuclear radius and ρ is an UV cutoff introduced to regulate the singular behavior at $x_\perp=0$. The extra factor of N_c in eq. (A.6) comes from calculating the color factor for non-Abelian energy-momentum tensor at the lowest order in the coupling. For simplicity, we consider a cylindrical nucleus such that $S_\perp \approx \pi\, A^{2/3}\, R_N^{2/3}$, with R_N the nucleon's radius. Introducing the dimensionful scale

$$\Lambda^2 \equiv \frac{4}{3\,\pi^3\, R_N^2}\, \ln\left(\frac{R}{\rho}\right), \tag{A.7}$$

we rewrite eq. (A.6) as

$$\langle T_{--}^{\,nucl}\rangle = N_c^2\, \alpha\, N_c\, 4\,\pi\, \Lambda^2\, A^{1/3}\, \frac{\delta(x^-)}{2\,|x^-|}. \tag{A.8}$$

Defining 't Hooft coupling λ by eq. (3.6) we rewrite eq. (A.8) as

$$\langle T_{--}^{\,nucl}\rangle = N_c^2\, \lambda\, \Lambda^2\, A^{1/3}\, \frac{\delta(x^-)}{2\,|x^-|}, \tag{A.9}$$

which is now ready to be cast in the form of eq. (3.5) at strong 't Hooft coupling by replacing $\lambda \to \sqrt{\lambda}$. The singularity in $\delta(x^-)/|x^-|$ can be regularized by replacing $1/|x^-|$ with the light-cone momentum of the valence gluon p^+, which would reduce eq. (A.9) to eq. (3.3) with μ given by eq. (3.4). The energy-momentum tensor in eq. (A.9) may serve as the source for the one-graviton exchange shock wave metric in eq. (3.1).

B. Solution of equations (4.9)

Here we complete the solutions of the Einstein equations for the second order correction to the metric, $g_{\mu\nu}^{(2)}$. Analogously to what we did for h in eq. (4.12), we write the unknown functions g, f and $\tilde{f}$ in eq. (4.8) in the form of a power series in z^2, starting at order z^4:

$$\begin{aligned} g(x^+,x^-,z) &= z^4 \sum_{n=0}^{\infty} g_n(x^+,x^-)\, z^{2n} \\ f(x^+,x^-,z) &= z^4 \sum_{n=0}^{\infty} f_n(x^+,x^-)\, z^{2n} \\ \tilde{f}(x^+,x^-,z) &= z^4 \sum_{n=0}^{\infty} \tilde{f}_n(x^+,x^-)\, z^{2n}\,. \end{aligned} \tag{B.1}$$

Inserting these expansions into eqs. (4.9e), (4.9f) and (4.9g) we find

$$g_n + 2h_n = \frac{2}{3}\, \delta_{n,2}\, t_1(x^-)\, t_2(x^+)\,, \tag{B.2}$$

$$f_{n,x^-} + h_{n,x^+} = -\,\frac{1}{12}\, \delta_{n,2}\, t_1(x^-)\, t_2'(x^+)\,, \tag{B.3}$$

$$\tilde{f}_{n,x^+} + h_{n,x^-} = -\,\frac{1}{12}\, \delta_{n,2}\, t_1'(x^-)\, t_2(x^+)\,, \tag{B.4}$$

which straightforwardly lead to the following relation between the metric coefficients

$$g(x^+,x^-,z) = -\,2\,h(x^+,x^-,z) + \frac{2}{3}\, z^8\, t_1(x^-)\, t_2(x^+)\,, \tag{B.5}$$

$$f(x^+,x^-,z) = -\,\frac{\partial_-}{\partial_+}\left(h(x^+,x^-,z) + \frac{1}{12}\, z^8\, t_1(x^-)\, t_2(x^+)\right), \tag{B.6}$$

$$\tilde{f}(x^+,x^-,z) = -\,\frac{\partial_+}{\partial_-}\left(h(x^+,x^-,z) + \frac{1}{12}\, z^8\, t_1(x^-)\, t_2(x^+)\right). \tag{B.7}$$

To complete our the solution of Einstein equations eqs. (4.9) we insert the relations in eqs. (B.5)–(B.7) into the remaining Einstein equations, eqs. (4.9a)–(4.9c), which we have not used yet. We find that, while eqs. (4.9a) and (4.9b) are trivially satisfied, eq. (4.9c) provides the additional non-trivial constrain for $h(x^+, x^-, z)$:

$$3\, h_z - z\, h_{zz} + 2\, z\, h_{x^+x^-} = \frac{8}{3}\, z^7\, t_1(x^-)\, t_2(x^+)\,, \tag{B.8}$$

which, using the general solution for $h(x^+, x^-, z)$ given in eq. (4.12), leads to

$$\frac{2}{\partial_+\partial_-}\left(I_2(z\sqrt{2\,\partial_+\partial_-}) - \frac{z}{8}\left(I_1(z\sqrt{2\,\partial_+\partial_-}) + I_3(z\sqrt{2\,\partial_+\partial_-})\right)\left(\sqrt{\frac{2}{\partial_+\partial_-}} + 2z\right)\right)$$
$$\left[h_0(x^+, x^-) - \frac{8}{(\partial_+\,\partial_-)^2}\, t_1(x^-)\, t_2(x^+)\right] = 0\,. \tag{B.9}$$

For arbitrary z, x^+ and x^-, this condition is satisfied only if the common coefficient of the different Bessel functions vanishes, i.e., if

$$(\partial_+\,\partial_-)^2\, h_0(x^+, x^-) \;=\; 8\, t_1(x^-)\, t_2(x^+). \tag{B.10}$$

This is exactly eq. (4.13) above. This condition makes the infinite series in eqs. (4.11) and (B.1) terminate, as previously announced. The complete solution of Einstein equations (4.9) is given by eqs. (4.18), (4.19), (4.20) and (4.22).

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